EFFECTIVE VISCOSITY AND HEAT CONDUCTIVITY OF A DISPERSED MEDIUM

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The effective viscosity and heat conductivity coefficients of dispersed media with spherically symmetric particles are calculated for various shapes of the binary distribution function.

Consider an isotropic dispersed medium, consisting of a continuous phase and solid spherical particles of radius a, whose bulk concentration is constant, randomly distributed in it. In the general case both phases can be mobile and have different velocities.

For such a system, satisfying all basic applicability conditions of the method of [1] and based on it, we calculate below the effective viscosity and heat conductivity coefficients, characterizing the momentum and heat transport, respectively. These coefficients appear in the equations describing transport processes for the medium as a whole.

Momentum Transport. Let the dispersed system be a suspension, whose continuous phase is a Newtonian fluid of viscosity μ_0 and density d_0. The Reynolds number, characterizing both local transport near the particles and the motion of the suspension as a whole, is assumed small in comparison with unity. The stationary motion of the suspension as a whole in the laboratory system of coordinates $\mathbf{r}$ is described by the mass and momentum conservation equations [1]

$$\nabla \mathbf{c} = 0;\ -\nabla p + \mu\Delta \mathbf{c} - d\nabla\Phi = 0;$$
$$\mathbf{c} = \varepsilon\mathbf{c}_0 + \rho\mathbf{c}_1;\ d = \varepsilon d_0 + \rho d_1;\ \rho = 1 - \varepsilon;\ \rho = \frac{4}{3}\pi a^3 n. \tag{1}$$

The effective viscosity coefficient μ appearing in (1) is determined from the self-consistency condition [1]

$$(\mu - \mu_0)\Delta \mathbf{v} = \nabla\left\{\rho\left[\frac{3}{4\pi a^3}\int\limits_{\mathbf{x}=\mathbf{a}} \mathbf{x} * \mathbf{n}\sigma(\mathbf{a}/0)\,d x + p\mathbf{I}\right]\right\}. \tag{2}$$

Here $\sigma(\mathbf{a}/0)$ is the tensor of conditional mean stresses acting on the surface $\mathbf{x} = \mathbf{a}$ through a separate ("trial") particle, and calculated by solving the flow problem of this particle in some fictitious homogeneous medium. The rheological properties of this medium when moving away from the particle surface coincide with the properties of the suspension as a whole, but at distances comparable with the particle radius they depend on distance.

We note that in the case under consideration the spatial derivatives of velocity in the laboratory and convective coordinate system $\mathbf{x}$, related to the center of the trial particle, coincide.

In determining the effective viscosity of the suspension the single-velocity model used here and the two-velocity model [2] give identical results. In calculating the sagging velocity of the suspension, however, which is beyond the scope of the present paper, it is necessary to use the two-velocity model.

Let the phase densities coincide $d_1 = d_0$, with the perturbation fields of velocity $\hat{\mathbf{v}}$ and $\hat{p}$ carried by the "trial" particle in the two-phase flow being determined by solving the problems [1,2]

$$\nabla \mathbf{v} = 0;\ -\nabla\hat{p} + 2\nabla\hat{\mu}\hat{\mathbf{e}} = 0;$$
$$\hat{\mathbf{v}} = -\mathbf{v} + \boldsymbol{\omega}\times\mathbf{x};\ \mathbf{x} = \mathbf{a};\ \hat{\mathbf{v}},\ \ \hat{p}\to 0;\ \mathbf{x}\to\infty;$$
$$\hat{\mu} = \mu_0 + (\mu - \mu_0)\psi(\xi,\ \rho);\ \xi = x/a;\ \hat{\mathbf{e}} = \frac{1}{2}\left\|\frac{\partial\hat{v}_i}{\partial x_j} + \frac{\partial\hat{v}_j}{\partial x_i}\right\|, \tag{3}$$

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where $\hat{e}$ is the velocity deformation tensor, and $\psi(\xi, \rho)$ characterizes features of particle distributions in the close neighborhood of an isolated particle. It can be shown* that the function $\psi(\xi, \rho)$ is uniquely determined by the binary distribution function (BDF)

$$\psi(\xi, \rho)=\rho^{-1}\int n(\mathbf{x}'/0)\,d\mathbf{x}';\ \ n(\mathbf{x}/0)=N\varphi(\mathbf{x}/0);\ |\mathbf{x}'-\mathbf{x}|\leqslant a,\ |\mathbf{x}'|>2a, \tag{4}$$

where $n(\mathbf{x}/0)$ is the conditional computed particle concentration.

Using the central symmetry, we obtain from (4)

$$\psi(\xi, \rho)=\begin{cases} -\dfrac{3n^{-1}}{4\xi}\displaystyle\int_{2}^{\xi+1} n(\eta,\ \rho)(\xi^2+\eta^2-1-2\xi\eta)\,\eta d\eta, & 1\leqslant\xi\leqslant 3; \\ -\dfrac{3n^{-1}}{4\xi}\displaystyle\int_{\xi-1}^{\xi+1} n(\eta,\ \rho)(\xi^2+\eta^2-1-2\xi\eta)\,\eta d\eta, & \xi>3. \end{cases} \tag{5}$$

For closure of problem (3) it is necessary to know the BDF appearing in (5). The determination of this function is a complicated problem of statistical physics and is possible only in the simplest of cases.

For opaque particles randomly arranged this problem was solved [3] in the superposition approximation, where it was shown that the BDF is an oscillating damped function, whose amplitude and amount of damping depend on the bulk concentration of the dispersed phase. The results of [3] were obtained under the assumption of chaotic particle arrangement.

The solution of the problem becomes more complicated for a suspension, since the binary distribution function depends on the flow parameters of the suspension, which in turn depend on the distribution function. Consequently, to determine the unknown function it is necessary to solve simultaneously an equation of type (3) and an equation describing the behavior of the BDF. For a quite dilute suspension, when it is sufficient to take into account pair interactions only, this problem was solved in [4]. In this case it was assumed that the suspension participates in purely linear shear motion. The BDF obtained in [4] was independent of the bulk concentration of the dispersed phase. This function is assumed given in the present paper.

Solution of the Test Particle Problem. To solve the problem (3) we use the method developed in [5], where we seek a solution of (3) in the form of series in basis vector functions, constructed on spherical functions

$$\hat{\mathbf{v}}=\sum_{m=0}^{\infty} f_m\frac{\mathbf{x}}{x}s_m+g_m x\nabla s_m;\ p=\frac{\mu_0}{a}\sum_{m=0}^{\infty} l_m s_m, \tag{6}$$

while the functions f_m, g_m, and l_m depend only on ξ and can be determined by the system of equations

$$\begin{gathered}\xi f_m'+2f_m-m(m+1)g_m=0,\\ [1+(\nu-1)\psi][\xi^2f_m''+2\xi f_m'-(2+m(m+1))f_m+2m(m+1)g_m]-\xi^2l_m'+2(\nu-1)\xi^2\psi'f_m'=0,\\ [1+(\nu-1)\psi][\xi^2g_m''+2\xi g_m'-m(m+1)g_m+2f_m]-\xi l_m+(\nu-1)\xi\psi'(\xi g_m'+f_m-g_m)=0.\end{gathered} \tag{7}$$

Here the prime denotes differentiation with respect to ξ and we have introduced the dimensionless effective viscosity coefficient $\nu = \mu/\mu_0$.

The unperturbed fields $\mathbf{v}$ and p can also be represented in the form of series of type (6) with functions F_m, G_m, and L_m. The coefficients of these functions are expressed in terms of values of the velocity $\mathbf{v}$ and its derivatives at the point occupied by the center of the test particle, and, consequently, can be considered as given a priori [5].

The boundary conditions for (7) are

$$f_m=-F_m,\ g_m=-G_m\quad \xi=1,\ f_m,\ g_m,\ l_m\to 0\quad \xi\to\infty. \tag{8}$$

*A detailed discussion of this problem is given, besides [1], in: Yu. A. Buevich, B. S. Endler, and I. N. Shchelchkova, "Continuum mechanics of multidispersed suspensions; rheological equations of state," Preprint No. 85, Inst. Prikl. Mekh. Akad. Nauk SSR, Moscow (1977).

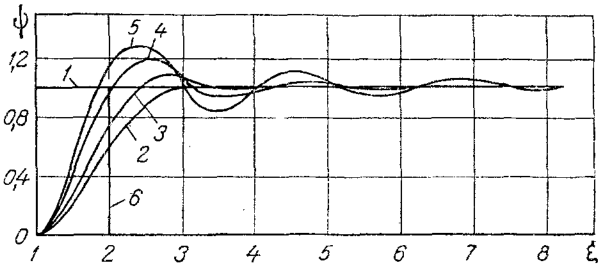

Fig. 1. Distance dependence of the function ψ: 1) interpenetrating particle model, 2) model using a stepwise BDF, 3) model using the BDF of [3] with $\rho = 0.144$; 4) 0.353; 5) 0.482; 6) model with a ring.

The solution of the problem (7), (8) completely determines the series (6) for the perturbation field and allows us to calculate the integral appearing in (2), which can be represented in the form

$$\frac{3}{4\pi a^3}\int\limits_{x=a} \mathbf{x} * \mathbf{n}\sigma(a/0)\,d\mathbf{x} = -p\mathbf{I} + \mu_0 5K^{(1)}\mathbf{e} + \mu_0 a^2 \frac{1}{2} K^{(2)}\Delta\mathbf{e}. \tag{9}$$

The coefficients $K^{(1)}$ and $K^{(2)}$ depend on ν, ρ and on the shape of the function $\psi(\xi, \rho)$ being used.

We note that due to the orthogonality of spherical functions only terms of the series (6) containing s_2 contribute to the surface integral (9). Therefore the lower subscript is henceforth omitted, being implied that it equals 2.

Taking into account that $\nabla(\Delta\mathbf{e}) = (1/2)\Delta^2\mathbf{v} \equiv 0$, we obtain from (2) and (9) an equation, determining the unknown parameter

$$\nu = 1 + \frac{5}{2}\rho K^{(1)}(\nu, \rho) \tag{10}$$

and the complete closure of the problem.

To solve (7), (8) numerically we transform them to the form

$$y - z^2 f'' = 0;\ A_1 y'' + A_2 y' + A_3 y + B_1 f' + B_2 f = 0;$$
$$A_1 = -[1 + (\nu - 1)\psi]\frac{z^2}{6};\ A_2 = (1 - \nu)\psi'\frac{z^2}{3};$$
$$A_3 = \frac{7}{3}[1 + (\nu - 1)\psi] + \frac{1}{6}(1 - \nu)\psi'' z^2;$$
$$B_1 = \frac{7}{3}(\nu - 1)\psi' z;\ B_2 = -4[1 + (\nu - 1)\psi] + \frac{2}{3}(1 - \nu)z(2\psi' + z\psi'');$$
$$f = f' = 0 \quad z = 0;\ f + F = 0,\ f' + 2F - 6G = 0 \quad z = 1, \tag{11}$$

where $z = \xi^{-1}$, the prime denotes differentiation with respect to z, and y(z) is a new function, determined by the first equation of (11). In deriving the boundary conditions we used the assumption that the continuity equation is valid everywhere up to the boundary.

To obtain the unknown dependence of ν on ρ the transcendental equation in ν (10), being in the given case

$$\nu = \{1 + \rho[(y'(1) + y(1))/30 - 1]\}(1 - \rho)^{-1},$$

was solved numerically by the Newton method. At each iteration step in ν the problem (11) was integrated by matrix methods so as to determine the value of the function y and its derivatives at z = 1.

Rheological Properties of Suspensions. As already mentioned, the BDF determines uniquely the function $\psi(\xi, \rho)$ appearing in (3).

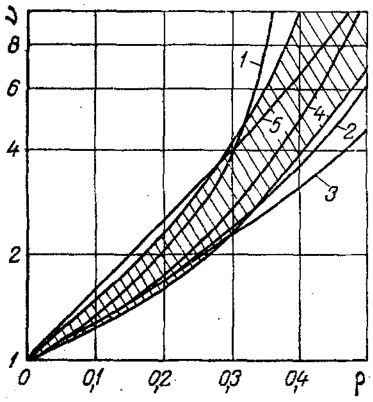

Fig. 2. Dependence of dimensionless suspension viscosity on bulk concentration: 1) interpenetrating particle model, 2) model using stepwise BDF, 3) model with a ring, Eq. (14), 4) BDF used in [3], 5) cellular model [6]; the primed region shows experimental data [7].

Neglecting the nonoverlap of particles, the BDF is constant and equal to unity in the whole interval $\xi \geqslant 1$. In that case $\psi = 1$ for $\xi \geqslant 1$ (Fig. 1, curve 1).

This assumption was used [1] in calculating the effective viscosity coefficient of moderately concentrated suspensions, and in [2] in determining the sedimentation velocity of these suspensions. The dependence obtained in [1] is represented graphically on curve 1 of Fig. 2.

Account of particle impenetrability leads to a more complicated form of the function $\psi(\xi, \rho)$. Using a BDF of the form

$$\varphi(\xi) = \begin{cases} 0 & 1 \leqslant \xi \leqslant 2, \\ 1 & \xi > 2, \end{cases}$$

we obtain from (5)

$$\psi = \begin{cases} \dfrac{27 - 56\xi + 30\xi^2 - \xi^4}{16\xi} & 1 \leqslant \xi \leqslant 3, \\ 1 & \xi > 3 \end{cases} \tag{12}$$

(Fig. 1, curve 2). We note that the function $\psi(\xi)$ of (12) is independent of the magnitude of the bulk concentration of the dispersed phase. The dependence of the dimensionless effective viscosity coefficient ν on the bulk concentration ρ, obtained as a result of numerical calculations with account of (12), is shown on Fig. 2, curve 2.

As a simplest approximation to (12) we use in the present paper the function

$$\psi(\xi) = \begin{cases} 0 & 1 \leqslant \xi \leqslant 2, \\ 1 & \xi > 2 \end{cases} \tag{13}$$

(Fig. 1, the broken line 6). In this case the problem (7), (8) has an analytic solution, whose substitution in (10) gives the following equation, determining the ρ-dependence of ν:

$$\nu = 1 + \rho \frac{0.7665\nu^3 + 6.0455\nu^2 + 5.9595\nu + 1.2285}{0.7665\nu^2 + 3.002\nu + 1.8315} \tag{14}$$

(Fig. 2, curve 3). Using the BDF of [3], the dependence $\psi(\xi, \rho)$ was obtained numerically in the present work (Fig. 1, curves 3-5). The results of calculating the ρ-dependence of ν, performed with the employment of this model, are shown in Fig. 2, curve 4.

For comparison Fig. 2 shows the ρ-dependence of ν, obtained in [6] on the basis of the cellular model.

The dashed region on Fig. 2 is formed by experimental data of sixteen papers, reviewed in [7]. The significant spread in the experimental data is related to the effect of many factors on measurement results: the type of viscosimeter used [8], the ratio of particle size to the size of flow region [8,9], the shear velocity [10], etc.

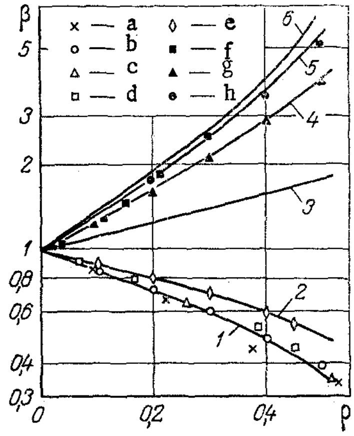

Fig. 3

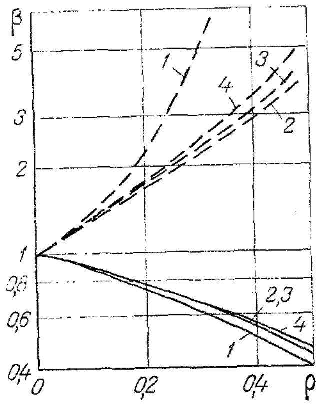

Fig. 4

Fig. 3. Dependence of the relative heat conductivity of a dispersed medium on bulk concentration: 1) at $\varkappa = 0$; 2) 0.172; 3) 3; 4) 15.7; 5) 100; 6) $\gg 1$: experimental data at $\varkappa = 0$: a) [12]; b) [13]; c) [14]; d) [15]; e) at $\varkappa = 0.172$; f) $\gg 1$; g) 15.7; h) 101.

Fig. 4. Dependence of the relative heat conductivity of a dispersed medium on bulk concentration at $\varkappa = 100$ (dashed) and $\varkappa = 0.1$ (full line) according to models: 1) of interpenetrating particles; 2) with a ring; 3) using a stepwise BDF [3]; and 4) results of the present work.

Comparison of the theoretical and experimental data shows that the model of impenetrable particles gives quite good results for moderately concentrated suspensions, but it enhances the value of the effective viscosity coefficient of suspensions of "mean" concentration, while for $\rho \to 40\%$ the value of the effective suspension viscosity becomes infinite.

Account of particle impenetrability becomes essential in determining the rheological characteristics of suspensions with higher than 20-25% concentration.

Heat Transport. Consider heat transport in a dispersed medium, both of whose phases can be mobile, under the assumption that the Peclet number, characterizing local convective transport near isolated particles of the dispersed phase flowing around the continuous phase, is small in comparison with unity. Obviously, the model under consideration is also valid for composite materials, whose matrix and dispersed phase have different heat conductivity coefficients.

According to [11], where references to related work can be found, the effective heat conductivity coefficient is represented in the form

$$\lambda = \lambda_0 \beta;\ \beta = 1 + (\varkappa - 1)\gamma;\ \varkappa = \lambda_1/\lambda_0, \tag{15}$$

where the unknown parameter γ is determined from the equation (compare with Eq. (2))

$$\gamma \mathbf{E} = \rho \left(\frac{4}{3}\pi a^3\right)^{-1} \int\limits_{\mathbf{x}=\mathbf{a}} \tau_1^* (\mathbf{a}/0)\, \mathbf{n} d\mathbf{x}, \tag{16}$$

while in determining the ensemble averaged of neighboring particles of temperature $\tau_1^*(\mathbf{a}/0)$ at the surface of the test particle (16) the following boundary-value problem is obtained:

$$\begin{gathered}\nabla [R(x/a)\nabla \hat{\tau}] = 0 \quad x > a;\quad \Delta \tau_1^* = 0 \quad 0 \leqslant x \leqslant a;\\ \hat{\tau} \to 0 \quad x \to \infty;\quad \tau_1^* < \infty \quad x = 0;\\ \hat{\tau} + \mathbf{E}\mathbf{x} = \tau_1^*;\quad \lambda_0 \mathbf{n} \nabla \hat{\tau} + \lambda \mathbf{n} \mathbf{E} = \lambda_1 \mathbf{n} \nabla \tau_1^* \quad x = a;\\ R = 1 + (\varkappa - 1)\gamma \psi(\xi, \rho).\end{gathered} \tag{17}$$

The function $\psi(\xi, \rho)$ is determined by the shape of the BDF according to (5).

Using the method discussed above of expanding the solutions in series of spherical functions we solved problem (17) in the present work, and determined the dependence of β on ρ and $\varkappa$ (Fig. 3). In the calculations we used the BDF obtained in [3] (see above).

As seen From Fig. 3, the dependence of the heat conductivity of a dispersed medium on the ratio of heat conductivities of the dispersed and continuous phases is most important at $\varkappa \lesssim 10$. For $\varkappa < 0.1$ and $\varkappa > 100$ the effective heat conductivity practically coincides with its values in the limiting cases $\varkappa \to 0$ and $\varkappa \to \infty$, i.e., when the dispersed phase consists of absolutely nonconducting or ideally conducting particles, respectively.

Using the fact that transport of heat, mass, or electric discharges under the approximations made is described by mathematically equivalent equations and, consequently, the dimensionless effective transport coefficients of these substances coincide, the results of calculations shown on Fig. 3 are equal to experimental data [12-15] on electric conductivity of stable emulsions. It is seen that the agreement between experiment and theory is quite good.

Comparison with data available in the literature on the effective heat conductivity of a stationary granular layer is not possible in the general case, since in the given paper no account is taken of heat transport in the dispersed phase due to particle contact.

To compare the various models we show in Fig. 4 a family of functions $\beta = \beta(\varkappa, \rho)$ at $\varkappa = 100$ and $\varkappa = 0.1$, obtained earlier under the following assumptions [11], similar to those made above: neglecting the nonoverlap of particles, curve 1; using the simplest representation of $\psi(\xi, \rho)$, taking into account particle impenetrability in the form (13), curve 2; $\psi(\xi)$ given in the form (12), curve 3. Also shown in Fig. 4 (curve 4) are results of calculations performed earlier.

It is seen by comparing Figs. 2 and 4 that the results of calculations, performed by using various BDF shapes and taking into account particle impenetrability (curves 2, 3, 4 on Figs. 2 and 4), give practically identical results for some effective heat conductivity. On the other hand, the values of the effective viscosity coefficient, as follows from Fig. 2, depends strongly on the BDF shape. In connection with the discussion above one must recall the large spread in experimental data of the effective viscosity coefficient of the suspension and the significantly smaller deviation between experimental data on effective heat conductivity coefficients. This is related to the structure of the dispersed phase, observed in real situations and leading to changes in the BDF shape.

NOTATION

Here a is the particle radius; A_i and B_i are coefficients in Eq. (11); **c** is the velocity in the laboratory coordinate system r; d is the density; **e** is the tensor of velocity deformations; f_m, F_m, g_m, G_m, l_m, L_m are functions in Eq. (6); **I** is the unit tensor; **E** is the mean density vector of thermal flux; $K^{(1)}$ and $K^{(2)}$ are coefficients in Eq. (9); m is the order of spherical harmonics; **n** is the unit vector of the outer normal to the surface of the test particle; N is the number of particles in the bulk; p is pressure; **v** is the velocity in the convective coordinate system **x**, referring to the center of the test particle; **x'** is the radius-vector of the particle center; y(z) is the function in Eq. (11); z is the independent variable in (11); β is the dimensionless effective heat conduction coefficient; γ is the coefficient in (15); ε is the porosity; η is the variable in (5); $\varkappa$ is the heat conduction ratio of dispersed and nondispersed phases; λ is the heat conduction coefficient; μ is the viscosity; ν is the dimensionless effective viscosity coefficient; ξ is the variable in (3) and later; ρ is the bulk concentration of the dispersed phase; σ is the stress tensor; τ is the temperature; Φ is the potential of external mass forces; φ is the binary distribution; ψ is the function defined in (4); ω is the angular velocity; the lower subscripts 0 refer to the continuous and dispersed phases, respectively; the asterisk denotes dyad multiplication; the overhead symbol denotes the perturbed field induced by the test particles; and the asterisk on top denotes temperature inside the test particle.

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